International Journal of Theoretical Physics, Vol. 31, No. 7, 1992

Equivalent Local Potentials for a Coupled-Channel System and the Feshbach Optical Potential

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Received March 28, 1991

Explicit formulas of all equivalent local potentials for a coupled n-channel problem are calculated. The general equivalent local potentials constitute a $[\binom{2n}{2}-1]$-complex-parameter family of local potentials. For a definite input elastic channel, the uniqueness of the equivalent local potential is shown. The equivalent local potential of the Feshbach optical potential coincides with the equivalent local potential of the n-channel system. The construction of the Feshbach optical potential is a reduction to the dimensionality of the coupled-channel problem, the construction of the equivalent local potential is a diagonalization of the coupled-channel problem, both constructions are compatible manipulations on the set of the coupled-channel system. The properties of the Feshbach optical potential can be used for the study of the properties of the equivalent local potential.

1. INTRODUCTION

Frequently a nonlocal potential is considered as a more complicated entity than a local one, and so different methods have been investigated in order to construct an *equivalent local potential* (ELP) to the n-channel Schrödinger equation with nonlocal interactions. The significance of the *equivalence* depends on the method of defining the ELP.

One method for the single-channel Schrödinger equation was proposed by Fiedeldey (1967). In this method the constructed ELP and the original nonlocal potential have a pair of solutions of the corresponding Schrödinger equations proportional to one function. This proportionality function is assumed to be equal to unity far from the range of the

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nonlocal potential. This assumption preserves the scattering properties (S-matrices, T-matrices, ...) of the original nonlocal potential. Each pair of independent solutions of the single-channel problem corresponds to a unique ELP. The Fiedeldey method was used with success in the nucleon–nucleon problems (Coz *et al.*, 1970), in the $\alpha\alpha$ interaction (Saito, 1987), and also in the case of the quark cluster interpretation of the NN interaction (Pantis, 1987). Mackellar and Coz (1976) studied a generalization of the Fiedeldey method concerning the coupled-channel problem. In the coupled-channel problem the number of solutions of the system of the Schrödinger equations is no longer two and there may exist more than a unique equivalent local potential. In Section 2 we investigate the number of possible ELPs.

The WKB method for the single-channel equation with a nonlocal potential was introduced by Horiuchi (1980). In this method the nonlocal potential is replaced by a local one, using the Wigner transform of the nonlocal operators. If the approximate equivalent local potential is treated by the WKB method, its WKB solutions are proportional to the WKB solutions of the original problem with the nonlocal potential. This method was successfully used for the quark structure investigations of the NN interaction by Shimizu (1989) and Suzuki and Hecht (1983). The WKB treatment of the coupled-channel equation with nonlocal potentials was introduced by Yabana and Horiuchi (1984). The WKB method of constructing the single-channel equivalent potential was generalized for the coupled-channel Schrödinger equation by Yabana and Horiuchi (1985*a*,*b*). In this method, the WKB solutions of the coupled-channel problem with nonlocal potentials are proportional to the WKB solutions of the equivalent system with local potentials. In these papers the WKB-equivalent coupled-channel problem contains local potentials which are linearly dependent on the momentum, if the nonlocal potentials are not symmetric.

In Section 2 we study the Fiedeldey–Mackellar–Coz method and give explicit expressions for the ELP in the elastic channel. In Section 3 we study the properties of the coupled-channel system and examine the relation between the Fiedeldey–Mackellar–Coz procedure and the Feshbach optical potential. The construction of the ELP is a local diagonalization of the nonlocal coupled-channel problem, while the Feshbach procedure is a reduction of the dimensionality of the coupled-channel system. This dimensional reduction can be applied iteratively and reduces the original coupled-channel problem to a single-channel nonlocal problem. We show that the ELP of the elastic channel coincides with the ELP of the reduced Feshbach optical potentials. So the Fiedeldey–Mackellar–Coz method of constructing ELP is compatible with the Feshbach optical potential. In Section 4, we summarize our result.

2. CONSTRUCTION OF THE EQUIVALENT LOCAL POTENTIAL

Let us consider now a coupled-channel problem:

$$\begin{aligned} a+A &\to a+A \\ &\to b+B \\ &\to c+C \\ &\to \dots \end{aligned}$$

The $a+A$ channel is called elastic and the rest of channels are treated as inelastic ones. This kind of interaction is described by a system of coupled integrodifferential equations:

$$\left(\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right)\psi_i(r)-\sum_{m=1}^{n}[V_{im}(r)-E_i\delta_{im}]\psi_m(r)$$
$$=\sum_{m=1}^{n}\int_0^\infty U_{im}(r,r')\psi_m(r')\,dr' \tag{1}$$

The Fiedeldey–Mackellar–Coz method consists in reducing this equation to an equivalent system of uncoupled n Schrödinger equations with local potentials. We start this section by giving a short account of this method, for the sake of clarity.

If $\{\mu_i(r)\}$ and $\{v_i(r)\}$, with $i=1,\dots,n$, are an independent pair of solutions of the system (1), the "Wronskians"

$$F_i(r)=f_i^2(r)=\mu_i' v_i-v_i'\mu_i$$

satisfy the system of equations

$$\frac{dF_i}{dr}=\sum_{m=1}^{n}V_{im}(r)[\mu_m(r)v_i(r)-v_m(r)\mu_i(r)]$$
$$+\sum_{m=1}^{n}\int_0^\infty U_{im}(r,r')[\mu_m(r')\mu_i(r)-v_m(r')\mu_i(r)]\,dr', \qquad i=1,\dots,n \tag{2}$$

We can define the functions

$$g_i(r)=\frac{\mu_i(r)}{f_i(r)} \qquad \text{and} \qquad h_i(r)=\frac{v_i(r)}{f_i(r)}, \qquad i=1,\dots,n \tag{3}$$

These functions constitute a pair of solutions for the system of equations

$$\left(\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right)\phi_i(r)+\chi_i(r)\frac{d\phi_i(r)}{dr}$$
$$=\sum_{m=1}^{n}[f_i^{-1}(r)V_{im}(r)f_m(r)-E_i\delta_{im}]\phi_m(r)$$
$$+\left[\frac{1}{2}\frac{F_i''}{F_i}-\frac{3}{4}\left(\frac{F_i'}{F_i}\right)^2\right.$$
$$\left.+\sum_{m=1}^{n}\int_0^\infty U_{im}(r,r')\frac{[\mu_i'(r)\nu_m(r')-\nu_i'(r)\mu_m(r')]\,dr'}{F_i}\right]\phi_i(r) \qquad (4)$$

where

$$\chi_i(r)=\sum_{m=1}^{n}V_{im}(r)\frac{\mu_i(r)\nu_m(r)-\nu_i(r)\mu_m(r)}{F_i} \qquad (5)$$

Equation (4) can be rewritten in a more compact form:

$$-\left(\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right)\phi_i(r)+\sum_{m=1}^{n}[V_{im}^{\mathrm{ELP}}(r,\hat{p})-E_i\delta_{im}]\phi_m(r)=0$$

where the potential $V_{im}^{\mathrm{ELP}}(r,\hat{p})$ is a momentum-dependent local operator:

$$V_{im}^{\mathrm{ELP}}(r,\hat{p})$$
$$=\left\{-\frac{d\chi_i(r)}{dr}+\frac{1}{2}\frac{F_i''}{F_i}-\frac{3}{4}\left(\frac{F_i'}{F_i}\right)^2+\frac{i}{2}[\chi_i(r)\circ\hat{p}+\hat{p}\circ\chi_i(r)]\right\}\delta_{im}$$
$$+f_i^{-1}(r)V_{im}(r)f_m(r)$$
$$+\sum_{k=1}^{n}\int_0^\infty U_{ik}(r,r')\frac{[\mu_i'(r)\nu_k(r')-\nu_i'(r)\mu_k(r')]\,dr'}{F_i}\delta_{im} \qquad (6)$$

where

$$\hat{p}\equiv\frac{1}{i}\frac{\partial}{\partial r}$$

The above local potential (6) contains diagonal momentum-dependent terms. Notice that in the case of the WKB method the equivalent local potentials are also momentum-dependent (Yabana and Horiuchi, 1985*a*,*b*), but the momentum terms appear in the nondiagonal coupling channel potentials $V_{im}^{\mathrm{WKB}}(r)$, $i\neq m$. Equation (4) can be reduced to a usual

Schrödinger equation without derivatives, even if the local potentials are not symmetric. We define the transformation

$$\psi_i(r) = \phi_i(r) \exp\left[\tfrac{1}{2} \int \chi_i(r)\, dr\right] \tag{7}$$

After the application of this transformation, the Fiedeldey–Mackellar–Coz theory for a coupled-channel problem can be summarized by the following proposition:

ELP for the Coupled-Channel Problem. If $\{\mu_i(r)\}$ and $\{v_i(r)\}$, $i = 1, \ldots, n$ are two independent solutions of equation (1), then the functions

$$u_i(r) = \exp\left[\tfrac{1}{2} \int \chi_i(r)\, dr\right] \mu_i(r)/f_i(r)$$

and

$$w_i(r) = \exp\left[\tfrac{1}{2} \int \chi_i(r)\, dr\right] v_i(r)/f_i(r), \qquad i = 1, \ldots, n$$

are two independent solutions of the Schrödinger coupled-channel system:

$$\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right)\psi_i(r) - \sum_{m=1}^{n} [V_{im}^{\mathrm{ELP}}(r) - E_i \delta_{im}]\psi_m(r) = 0 \tag{8}$$

where $V_{im}^{\mathrm{ELP}}(r)$ is given by the following formula:

$$\begin{aligned} V_{im}^{\mathrm{ELP}}(r) &= \frac{\exp[\frac{1}{2}\int \chi_i(r)\, dr]}{f_i(r)} V_{im}(r) \exp\left[-\frac{1}{2}\int \chi_m(r)\, dr\right] f_m(r) \\ &+ \left[\frac{1}{2}\frac{d\chi_i}{dr} + \frac{1}{2}\frac{F_i''}{F_i'} - \frac{3}{4}\left(\frac{F_i'}{F_i}\right)^2\right]\delta_{im} \\ &+ \int_0^\infty \sum_{k=1}^{n} U_{ik}(r, r') \frac{\mu_i'(r) v_k(r') - v_i'(r)\mu_k(r')}{F_i(r)}\, dr'\, \delta_{im} \end{aligned} \tag{9}$$

After a little algebra we find

$$\begin{aligned} \mathrm{V}_{im}^{\mathrm{ELP}}(r) &= \frac{\exp[\frac{1}{2}\int \chi_i(r)\, dr]}{f_i(r)} V_{im}(r) \\ &\times \exp\left[-\frac{1}{2}\int \chi_m(r)\, dr\right] f_m(r) + V_i^{\mathrm{diag}}(r)\delta_{im} \end{aligned} \tag{10}$$

where

$$V_i^{\text{diag}}(r) = -\frac{3}{4}\left(\frac{F_i'}{F_i}\right)^2 - \frac{1}{2}\int_0^\infty \sum_{k=1}^n U_{ik}(r, r') \times \frac{\mu_i'(r)\nu_k(r') - \nu_i'(r)\mu_k(r')}{F_i(r)} + \frac{1}{2}\int_0^\infty \sum_{k=1}^n \frac{\partial U_{ik}(r, r')\mu_i(r)\nu_k(r') - \nu_i(r)\mu_k(r')}{F_i(r)} \tag{11}$$

The nondiagonal term in the ELP (10) can be eliminated by redefining the initial potentials as follows:

$$\begin{aligned} \hat{V}_{im}(r) &= 0 \\ \hat{U}_{im}(r, r') &= U_{im}(r, r') + V_{im}(r)\delta(r - r') \end{aligned} \tag{12}$$

This ELP was studied by Mackellar and Coz (1976).

The most usual coupled channel problem is the one without nonlocal terms:

$$\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right)\psi_i(r) - \sum_{m=1}^n [V_{im}(r) - E_i\delta_{im}]\psi_m(r) = 0 \tag{13}$$

In this case the explicit form of the ELP is given by the following proposition:

ELP for a System of Coupled Local Equations. Let $\mu_i(r)$ and $\nu_i(r)$ be two independent solutions of the system (13), with local potentials. Then we define

$$F_i(r) = \begin{vmatrix} \mu_i'(r) & \mu_i(r) \\ \nu_i'(r) & \nu_i(r) \end{vmatrix}, \qquad i = 1, \ldots, n \tag{14}$$

where the functions $u_i(r)$ and $w_i(r)$ are defined as follows:

$$u_i(r) = \frac{\mu_i(r)}{[F_i(r)]^{1/2}} \quad \text{and} \quad w_i(r) = \frac{\nu_i(r)}{[F_i(r)]^{1/2}} \tag{15}$$

are two independent solutions of the uncoupled Schrödinger equation:

$$-\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right)\psi_i(r) + [V_i^{\text{ELP}}(r) - E_i]\psi_i(r) = 0 \tag{16}$$

where $V_i^{\mathrm{ELP}}(r)$ is the ith channel ELP, given by the formula

$$V_i^{\mathrm{ELP}}(r) = -\frac{3}{4} \sum_{m=1}^{n} V_{im}(r) \frac{\begin{vmatrix} \mu_i(r) & \mu_m(r) \\ v_i(r) & v_m(r) \end{vmatrix}^2}{F_i(r)}$$

$$+\frac{1}{2} \sum_{m=1}^{n} \frac{dV_{im}}{dr} \frac{\begin{vmatrix} \mu_i(r) & \mu_m(r) \\ v_i(r) & v_M(R) \end{vmatrix}}{F_i(r)}$$

$$+ \sum_{m=1}^{n} V_{im}(r) \frac{\left(\begin{vmatrix} \mu_i(r) & \mu_m'(r) \\ v_i(r) & v_m'(r) \end{vmatrix} - \begin{vmatrix} \mu_i'(r) & \mu_m(r) \\ v_i'(r) & v_m(r) \end{vmatrix} \right)}{F_i(r)} \tag{17}$$

These formulas are the explicit forms of the ELP for the local n-channel problem.

The number of the independent solutions of the system (1) or (16) is 2^n. From equation (17) we conclude that for each pair of linear independent solutions

$$\begin{bmatrix} \mu_i(r) \\ v_i(r) \end{bmatrix}_{i=1,\ldots,n} \tag{18}$$

we can find the ELP $V_i^{\mathrm{ELP}}(r)$, $i = 1, \ldots, n$, given by equation (17). The solutions (18) are solutions of the coupled-channel problem uniquely determined by appropriate boundary conditions. Thus, each ELP depends on the boundary conditions imposed on the initial pair of the solutions (18). These conditions mean a definition of the initial input elastic channel and asymptotic output conditions for the inelastic channels, as we shall see.

We can choose $\binom{2^n}{2}$ pairs of independent solutions of the form (18), because the number of the independent solutions of the system (1) is 2^n. These independent pairs can be marked as follows:

$$\begin{bmatrix} \mu_i^p(r) \\ \omega_i^p(r) \end{bmatrix}_{i=1,\ldots,n}, \qquad p = 1, 2, \ldots, \binom{2^n}{2} \tag{19}$$

The general solution of the system (1) can be written as follows:

$$\begin{bmatrix} \mu_i(r) \\ v_i(r) \end{bmatrix}_{i=1,\ldots,n} = \sum_{p=1,\ldots,\binom{2^n}{2}} a_p \begin{bmatrix} \mu_i^p(r) \\ v_i^p(r) \end{bmatrix}_{i=1,\ldots,n} \tag{20}$$

where a_p are complex numbers.

The general form of the ELP can be obtained by replacing the μ's and ν's in equation (17) by those given by relation (20). All the components of the ELP in equation (14) contain ratios of determinants; thus, without loss of generality we can put $a_1 = 1$ and make the following statement.

Proposition 1. The set of possible equivalent potentials is parametrized by $\binom{2n}{2} - 1$ independent complex parameters a_p, $p = 2, \ldots, \binom{2n}{2}$.

In the case of the single-channel problem system ($n = 1$), two independent solutions exist, so there is only one ELP, not depending on the initial conditions. In the case of the multi-channel problems, the situation appears more complicated.

Usually the coupled-channel Schrödinger equation describes the reactions with many output (inelastic) channels but only one input (elastic) channel. The particles in an (output) inelastic channel move out the center of the interaction and they behave as independent particles after a sufficiently large time. Let $i = 1$ be the elastic channel and $i = 2, \ldots, n$ be the output inelastic channels. In this case the boundary conditions imposed on all the inelastic channels are

$$\psi_i(r) \sim \exp(ik_i r) \quad \text{if} \quad r \to \infty, \qquad k_i = \sqrt{E_i}, \qquad i = 2, \ldots, n \tag{21}$$

If the above restriction is imposed, the number of the independent solutions of the system (1) is exactly two. If there are Coulomb potentials in the channel potentials $V_{ii}(r)$, then the exponential function in equation (1) must be replaced by the corresponding asymptotic form of the Coulomb function. Therefore we can make the following statement.

Proposition 2. In the multichannel problem

$$\begin{aligned} a + A &\to a + A \\ &\to b + B \\ &\to c + C \\ &\to \ldots \end{aligned}$$

for a given elastic channel $a + A$, *only one* ELP, corresponding to outgoing waves in the inelastic channels ($b + B$, $c + C, \ldots$), can be found.

The asymptotic condition (21) could be satisfied if the coupling potentials $V_{im}(r)$, $U_{im}(r, r')$, $i \neq m$, have a finite range R, which means that these potentials are zero for $r > R$. In the theory of nuclear interactions the basic interactions are the Coulomb interaction combined with the strong interaction between nucleons. The Coulomb interaction does not have a finite range, but it is present only in the diagonal channel local potentials $V_{ii}(r)$.

The coupling potentials are derived from combinations of microscopic nucleon–nucleon potentials which have a finite range. In this case, for $r > R$, the initial system (1) is written in a uncoupled form. Solutions satisfying the condition (21) exist and the method can be applied, but all the definite integrals in equation (11) should be calculated from 0 to R; this equation (11) is valid when $r < R$. For $r > R$ the ELP are equal to the local parts of the channel potentials.

We can give a formulation of the Fiedeldey–Mackellar–Coz method useful for further applications. Let Ω_n be the set of all n-channel equations like equation (1); Ω_n^{loc} can be defined as the subset of Ω_n containing the systems of n equations with local kernels like equation (8). For every system x in Ω_n, the linear 2^n-dimensional space of the solutions of the coupled differential equations (1) is attached. For two fixed solutions $\mu_i(r)$ and $v_i(r)$ of the system x, the Fiedeldey–Mackellar–Coz method is an application from Ω_n into Ω_n^{loc},

$$\Omega_n \ni x \to \text{FMC}(x) \in \Omega_n^{\text{loc}}$$

The equivalent local system FMC(x) has two solutions $\{u_i(r)\}$ and $\{w_i(r)\}$ with constant Wronksians $W[u_i, w_i] = 1$ for every i.

The set of the n uncoupled systems in this formulation is

$$[\Omega_1^{\text{loc}}]^n = \underbrace{\Omega_1^{\text{loc}} \times \Omega_1^{\text{loc}} \times \ldots \Omega_1^{\text{loc}}}_{n \text{ times}}$$

Any uncoupled local system does not change when the Fiedeldey procedure is applied; therefore

$$\text{FMC}([\Omega_1^{\text{loc}}]^n) = [\Omega_1^{\text{loc}}]^n \tag{22}$$

3. THE FESHBACH OPTICAL POTENTIAL

An interesting topic is the relation between the Fiedeldey method and the Feshbach procedure of constructing the optical potential of a coupled channel system (Feshbach, 1967). We consider the system

$$\begin{aligned}\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right)\psi_i(r) - [V_{ii}(r) - E_i]\,\psi_i(r)\\ - \int_0^\infty U_{ii}(r, r')\,\psi_i(r')\,dr'\\ = \sum_{\substack{m=1\ldots n\\ m\neq i}} \left\{[V_{im}(r) - E_i\delta_{im}]\,\psi_m(r) + \int_0^\infty U_{im}(r, r')\,\psi_m(r')\,dr'\right\}\end{aligned} \tag{23}$$

Let $G_n(E_n, r, r')$ be the Green's function of the nth equation

$$\left(\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right)G_n(E_n, r, r')-[V_{nn}(r)-E_n]G_n(E_n, r, r') - \int_0^\infty U_{nn}(r, r')\psi_n(r')\,dr' = \delta(r-r') \tag{24a}$$

The nth equation in system (23) can be solved:

$$\psi_n(r) = \sum_{k=1}^{n-1}\int_0^\infty dr'\, G_n(E_n, r, r')V_{nk}(r')\psi(r') + \sum_{k=1}^{n-1}\int_0^\infty dr' \int_0^\infty dr''\, G_n(E_n, r, r'')U_{nk}(r'', r')\psi(r') \tag{24b}$$

After replacing ψ_n in all equations, we arrive at the optical system, with $n-1$ equations:

$$\left(\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right)\psi_i(r)-[V_{ii}(r)-E_i]\psi_i(r) - \int_0^\infty U_{ii}(r, r')\psi_i(r')\,dr' = \sum_{\substack{m=1\ldots n-1\\ m\neq i}}\left\{[V_{im}(r)-E_i\delta_{im}]\psi_m(r) - \int_0^\infty \hat{U}_{im}(r, r')\psi_m(r')\,dr'\right\} \tag{25}$$

where the potential $\hat{U}_{ik}$ is defined as follows:

$$\hat{U}_{ik}(r, r') = U_{ik}(r, r') + V_{in}(r)G_n(E_n, r, r')V_{nk}(r') + V_{in}(r)\int_0^\infty dw\, G_n(E_n, r, w)U_{nk}(w, r') + \int_0^\infty ds\, U_{in}(r, s)G_n(E_n, s, r')V_{nk}(r') + \int_0^\infty ds \int_0^\infty dw\, U_{in}(r, s)G_n(E_n, s, w)U_{nk}(w, r') \tag{26}$$

We notice that the initial system (23) of n equations is reduced to a system with $n-1$ equations (25). If we repeat this procedure $n-1$ times, we find the Feshbach local potential as described by Feshbach (1967). Symbolically, this reduction of the rank is expressed by the application

$$F: \quad \Omega_n \ni x \to F(x) \in \Omega_{n-1} \tag{27}$$

where $F(x)$ represents the system derived by the Feshbach procedure. The exact definition of this function presupposes the choice of the pair $\{\mu_i(r)\}$ and $\{v_i(r)\}$, $i=1, \ldots, n$. This choice defines uniquely the channel Green's functions $G_i(E_i, r, r')$ with the appropriate boundary conditions. The solutions $\{\mu_j(r)\}$ and $\{v_j(r)\}$, $j=1, \ldots, n-1$, of both systems coincide for the first $n-1$ channels.

An interesting property of the Fiedeldey–Mackellar–Coz method is its compatibility with the Feshbach reduction scheme. The following proposition is true:

Proposition 3. For every system $x \in \Omega_n$ and for a given choice of a pair of solutions $\{\mu_i\}$, $\{v_i\}$ the following relation is true:

$$\mathrm{FMC}(x) = \mathrm{FMC}(F(x)) \tag{28}$$

We consider now the case with nonlocal potentials, i.e., $V_{im}=0$, as we have shown in (12) that this case is the general one. The first $n-1$ solutions $\psi_j(r)$, $j=1, \ldots, n-1$, of the system (23) (which is a system in Ω_n) are solutions of the system (25) (which is a system in Ω_{n-1}). Consequently, the corresponding Wronksians coincide:

$$F_i(r) = \hat{F}_i \qquad \text{for} \quad i=1, \ldots, n-1$$

It can be seen that

$$\begin{aligned}
&\int_0^\infty \sum_{m=1}^{n-1} \hat{U}_{im}(r, r')[\mu_i'(r) v_m(r') - v_i'(r)\mu_m(r')]\, dr' \\
&= \int_0^\infty \sum_{m=1}^{n-1} U_{im}(r, r')[\mu_i'(r) v_m(r') - v_i'(r)\mu_m(r')]\, dr' \\
&\quad + \sum_{m=1}^{n-1} \int_0^\infty dr' \int_0^\infty ds \int_0^\infty dw\; U_{in}(r, s) G_n(E_n, s, w) \\
&\quad \times U_{nm}(w, r')[\mu_i'(r) v_m(r') - v_i'(r)\mu_m(r')] \\
&= \int_0^\infty \sum_{m=1}^{n} U_{im}(r, r')[\mu_i'(r) v_m(r') - v_i'(r)\mu_m(r')]\, dr'
\end{aligned}$$

From equation (8′) we conclude that the ELPs in both cases are the same.

An obvious generalization of Proposition 3 is the following corollary:

Corollary. For every system $x \in \Omega_n$ the following equality is valid:

$$\mathrm{FMC}(x) = \mathrm{FMC}(F^p(x)) \tag{29}$$

This proposition means that the Fiedeldey–Mackellar–Coz procedure, applied to the Feshbach optical potential, creates an ELP which is the same as the ELP derived by the (uncoupled) system. We can say that the Feshbach and Fiedeldey–Mackellar–Coz procedures are compatible manipulations on the differential systems. Thus, the properties of the Feshbach optical potentials can be used for the study of the properties of the complicated ELPs given by equation (17). The Feshbach optical potential has been constructed by using the Green's operators:

$$G_k = \frac{1}{E_k - H_k}$$

These Green's operators converge to zero for great values of the energies E_k or for large values of the angular momenta l; therefore we have the following results.

Proposition 4. The ELPs $V_i^{\mathrm{ELP}}(r)$ given by equation (17) converge to the channel potentials $V_{ii}(r)$ when the energies E_i or the angular momenta l take great values.

The properties of the ELPs for a coupled-channel square-well potential are under investigation and numerical calculations will be forthcoming. These preliminary calculations show that the ELPs are smooth potentials without singularities due to the existence of the Wronskians $F_i(r)$ in the denominators of equation (17). This smooth character of the ELP of the coupled-channel system indicates that the ELP can be used for the study of the multichannel problem, but the assumed smoothness of the ELP should be proven mathematically.

4. SUMMARY

In this paper we study the Fiedeldey–Mackellar–Coz method for the coupled-channel case. For every pair of solutions of the coupled-channel problem we can define equivalent potentials such that the coupled-channel problem with nonlocal potentials is transformed into a coupled problem with local potentials. The ELPs in the coupled-channel case depend on the boundary conditions imposed on the solutions of the exact problem. In the case of the coupling potentials with finite range we can construct one ELP for the case of outgoing inelastic wave functions. We give the explicit formula for the ELP in the coupled-channel case. The Fiedeldey–Mackellar–Coz method is a procedure for transforming a coupled-channel system to an uncoupled one. The Feshbach optical potential method is a procedure to reduce the rank of the system. In this paper we proved that the Fiedeldey method is related to the Feshbach procedure. The two methods can be viewed as compatible manipulations on differential systems.

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